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# Gauge transformations and vacuum structure in the Schwinger model*

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Following recent proposals we reexamine the vacuum structure of the Schwinger model in terms of topologically inequivalent gauge classes.

Considerable insight has recently been obtained into the vacuum structure of gauge theories[1,2,3] on the basis of the pseudoparticle solution[4] considered as a link between inequivalent gauge classes. It has been pointed out in Refs. 2 and 3 that the vacuums are characterized by a chiral angle associated with the spontaneous breakdown of the corresponding symmetry, in complete analogy with what is known to occur[5] in the Schwinger model.[6] Following the analysis presented in Refs. 2 and 3 for the Abelian case, this similarity with the Schwinger model appears at first sight puzzling since we do not have here any Higgs fields to support the Nielsen-Olesen vortex,[7] which plays the role of the pseudoparticle in two-dimensional space-time. It is the purpose of this comment to clarify the existence of inequivalent gauge classes and their chiral quantum numbers in the Schwinger model by providing an explicit representation of the operators, implementing gauge transformation of the second kind, and connecting them with the vacuum-raising operators of Ref. 5.

It is clear that, in order to have a unitary representation of a gauge transformation of the second kind, we should start with the formulation of the Schwinger model in an indefinite-metric "Hilbert" space. Following Ref. 5 we have for the fermion field $\psi$ and electromagnetic potential $A_\mu$ the following expressions:

$$\psi(x) = :\exp\{i\sqrt{\pi}\,\gamma^5[\tilde{\eta}(x) + \tilde{\Sigma}(x)]\}:\psi_0(x)\,,$$

$$A^\mu(x) = -\frac{\sqrt{\pi}}{e}\epsilon^{\mu\nu}\partial_\nu[\tilde{\Sigma}(x) + \tilde{\eta}(x)]\,,$$

where $\psi_0(x)$ is the free canonical massless fermion field, $\tilde{\Sigma}(x)$ is a free pseudoscalar field of mass $e/\sqrt{\pi}$, and $\tilde{\eta}(x)$ is a zero-mass gauge excitation quantized with indefinite metric. This corresponds to an operator realization of the original Schwinger solution[6] in the Lorentz gauge. It is easily checked that the operator $T[\Lambda]$, which implements the gauge transformation

$$T\psi(x)T^{-1} = e^{i\Lambda(x)}\psi(x),$$

$$TA^\mu(x)T^{-1} = A^\mu(x) + \frac{1}{e}\partial^\mu\Lambda(x)\,,$$

is given by

$$T[\Lambda] = \exp\left(\frac{i}{\sqrt{\pi}}\int_{y^0} dy^1\{[\tilde{\eta}(y) + \tilde{\phi}(y)]\partial_1\Lambda(y) - [\eta(y) + \phi(y)]\partial_0\Lambda(y)\}\right)\,, \tag{1}$$

where $\phi$ and $\tilde{\phi}$ are the potentials of the free current and pseudocurrent, respectively,

$$j_f^\mu = -\frac{1}{\sqrt{\pi}}\partial^\mu\phi\,, \quad j_{f5}^\mu = \epsilon^{\mu\nu}j_{f\nu} = -\frac{1}{\sqrt{\pi}}\partial^\mu\tilde{\phi}\,,$$

and

$$\partial_\mu\eta = \epsilon_{\mu\nu}\partial^\nu\tilde{\eta}\,.$$

Since we are working in the Lorentz gauge, $\Lambda$ must be a solution of the wave equation. This at the same time ensures the time independence of the operator, Eq. (1). It is important to notice at this stage that the right-hand side of Eq. (1) is a well-defined operator only as long as

$$\Lambda(x^0, x^1) \xrightarrow[|x^1|\to\infty]{} 0\,,$$
$$\int dy^1\partial_0\Lambda(y^0, y^1) = 0\,, \tag{2}$$

which means, since $\Lambda(y)$ is a solution of the wave equation

$$\Lambda(x^0, x^1) = g(x^1 - x^0) + h(x^1 + x^0)\,, \tag{3a}$$

that

$$g(\pm\infty) = h(\pm\infty) = 0\,. \tag{3b}$$

This is necessary since, whereas $\tilde{\eta}(x)$ has been quantized with an indefinite metric and therefore is defined for arbitrary test functions, the free current potentials $\phi(x)$ and $\tilde{\phi}(x)$ only have meaning in their Fock space if smeared with test functions whose total integral vanishes, i.e., Eqs. (2).[8] If we want to consider a broader class of gauge transformation, such as the ones discussed in Refs. 2 and 3, we need to relax condition (2) and are led to a representation of the current potentials inequivalent to the Fock representation.[9] For instance, let us consider a solution $\Lambda_1$ of the wave equation satisfying the initial conditions

$$\Lambda_1(0,-\infty)=0,\quad \Lambda_1(0,+\infty)=2\pi,\quad \dot{\Lambda}_1(0,x^1)=0\,.$$

It is readily checked that the operator $T[\Lambda_1]$ has the following properties:

(a) $\langle 0|T[\Lambda_1]|0\rangle=0$,

(b) $\langle\Psi|T^*[\Lambda_1']T[\Lambda_1'']|\Phi\rangle=\langle\Psi|\Phi\rangle$,
where $|\Psi\rangle$ and $|\Phi\rangle$ are states in the gauge-invariant subspace defined by

$$\langle\Psi|\partial^\mu(\eta+\phi)|\Phi\rangle=0\,,$$

and

(c) $T\tilde{Q}_5T^{-1}=\tilde{Q}_5+2$,

where $\tilde{Q}_5$ is the space integral of the time component of the conserved gauge-noninvariant $\gamma_5$ current $j^\mu_{f5}(x)$.

Property (a) follows from the fact that the operator $T[\Lambda_1]$ leads one to an inequivalent representation of the current potentials; it acts as a vacuum-raising operator. Property (b) expresses the fact that the space-time dependence of $\tilde{\phi}(x)$ is compensated by the one of $\tilde{\eta}(x)$ on the gauge-invariant subspace; the only reason for $T[\Lambda_1]$ not being the identity operator is that the exponential of $\tilde{\phi}$ carries a selection rule.[10] Property (c) is to be expected from Refs. 1, 2, and 3 and it identifies the selection rule as being twice the chiral quantum number. The above properties are shared by the $\sigma_1^*\sigma_2$ vacuum-raising operator introduced in Ref. 5. In fact, using the Bose form for the free fermion field[10,11] one has in the original Schwinger gauge

$$\sigma_1^*\sigma_2=\exp\{2i\sqrt{\pi}\,[\tilde{\eta}(x)+\tilde{\phi}(x)]\}\,. \tag{4}$$

By writing $\Lambda_1$ in the form

$$\Lambda_1(y^0,y^1)=\overline{\Lambda}(y^0,y^1)+2\pi\theta(y^1-x^1)\,,$$

where $\overline{\Lambda}$ generates the identity transformation on the gauge-invariant subspace, we are immediately led to the identification of the operator (4) with $T[\Lambda_1]$.

Up to now we have considered only gauge transformations which satisfy the criterion of Ref. 2, $\exp[i\Lambda(x^0,\pm\infty)]=1$. This gave us the connection with $\sigma_1^*\sigma_2$. However, by relaxing this condition we can also relate the Bose form of $\sigma_1$ and $\sigma_2$,

$$\sigma_\alpha=\exp\bigl(i\sqrt{\pi}\,\{[\eta(x)+\phi(x)]+(-1)^\alpha[\tilde{\eta}(x)+\tilde{\phi}(x)]\}\bigr) \tag{5}$$

to the operators $T[\Lambda_{+1/2}]$ and $T[\Lambda_{-1/2}]$, respectively, where $\Lambda_{\pm1/2}$ are given by

$$\Lambda_{+1/2}(x^0,x^1)=g(x^1-x^0)\,,$$

$$\Lambda_{-1/2}(x^0,x^1)=h(x^1+x^0)\,,$$

with

$$g(-\infty)=h(-\infty)=0\,,$$

$$g(+\infty)=-h(+\infty)=\pi\,.$$

The fact that

$$\exp[i\Lambda_{\pm1/2}(x^0,\infty)]=-1$$

seems to signalize the spin-$\frac{1}{2}$ nature of the underlying canonical fermion field. Thus by introducing this broader class of gauge transformation one can exhibit the complete vacuum structure of the Schwinger model[5]:

$$(\sigma_1)^{n_1}(\sigma_2)^{n_2}|0\rangle=|n_1,n_2\rangle\,.$$

The operators $\sigma_\alpha$ carry the whole free charge in this model

$$\tilde{Q}=\int dx^1 j^0_f(x)\,,$$

thus preventing the appearance of charged physical states (confinement). Contrary to what happens with the class $\Lambda_1$ of gauge transformations, the bigger class, including $\Lambda_{\pm1/2}$ leading to the $\sigma_\alpha$, cannot be obtained from the trivial class by means of the pseudoparticle in the manner explained in Refs. 2 and 3. Is this an indication that the vacuum structure of confining gauge theories is even richer than the one expected from the topological considerations of Refs. 1–4?

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